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Mode and Transducer Selection for Long Range Lamb Wave Inspection

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ABSTRACT: Lamb waves can propagate many metres along plate and shell structures, and so have great potential in 'smart structure' applications where it is important for a transducer to interrogate a significant area of the surrounding structure. However, there are many different types of Lamb wave and in order to obtain simple signals that can be reliably interpreted, it is important to excite a single mode in a well controlled direction. The choice of which Lamb wave mode to use in a particular application depends on numerous factors, including the resolution required, the type of defects or damage to be detected, the attenuation and the available transduction options. This paper sets out a rational procedure for identifying suitable Lamb wave modes and operating frequencies for a particular inspection task. It is shown that the properties of the system to be inspected determine which mode and frequencies can be used, and that this then dictates the type of transducer required. A procedure for evaluating the performance of Lamb wave transducers is also demonstrated. As an illustrative example, it is shown that the well known angle incidence transduction technique is not generally suitable in applications where the structure to be inspected is liquid loaded. In such cases it is necessary to consider alternative transduction options such as electromagnetic acoustic transducers (EMATs) or shear piezoelectric devices.

INTRODUCTION

CONVENTIONAL ultrasonic inspection of plate-like structures requires a probe to be scanned over the whole area to be tested. In the simplest case, Lamb wave inspection enables a line of such a structure to be tested from a single location. Hence a two-dimensional scan using conventional ultrasonic techniques can be reduced to a one-dimensional scan using Lamb waves. In such an application movable transducers may be used and the benefit of using Lamb waves is to reduce the time and cost of inspection. In order to create a 'smart' or 'self inspecting' structure, transducers must be permanently attached or integrated into the structure. In order to inspect an entire structure using permanently attached ultrasonic transducers there are two possibilities. The first option is to use a very large number of transducers that individually only inspect the area of the structure immediately beneath them. Because of the overall cost, the complexity of connections, and the fact that in many cases the instrumented structure would no longer be able to fulfil its original function, this option is generally impractical. The only remaining option is to use a smaller number of transducers that are individually capable of monitoring a significant area of the surrounding structure. In this scenario, the use of Lamb waves (or other types of guided waves) becomes a necessity.

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The main problems with using Lamb waves are that there are an infinite number of different modes that can propagate and all of the modes are dispersive. This can be readily seen when one considers the Lamb wave phase velocity, v_{ph}, and group velocity, v_{gr}, dispersion curves shown in Figure 1 for a steel plate. These curves were plotted using the software suite *Disperse* (Pavlakovic et al., 1997). By plotting the product of frequency, f, and plate thickness, d, on the abscissa, these curves can be used for any thickness of plate.

In an inspection application where Lamb waves are used in either a pulse-echo or pitch-catch configuration, it is necessary to operate with a single Lamb wave mode over a limited frequency range (Alleyne and Cawley, 1992a). The question which then arises is which mode to use and over what frequency range. The aim of this paper is to set out a rational procedure which enables these questions to be answered for a particular inspection task.

The basic factors which determine which Lamb wave mode and frequency to use may be enumerated as follows:

(i) dispersion,
(ii) attenuation,
(iii) sensitivity,
(iv) excitability,
(v) detectability and
(vi) selectivity.

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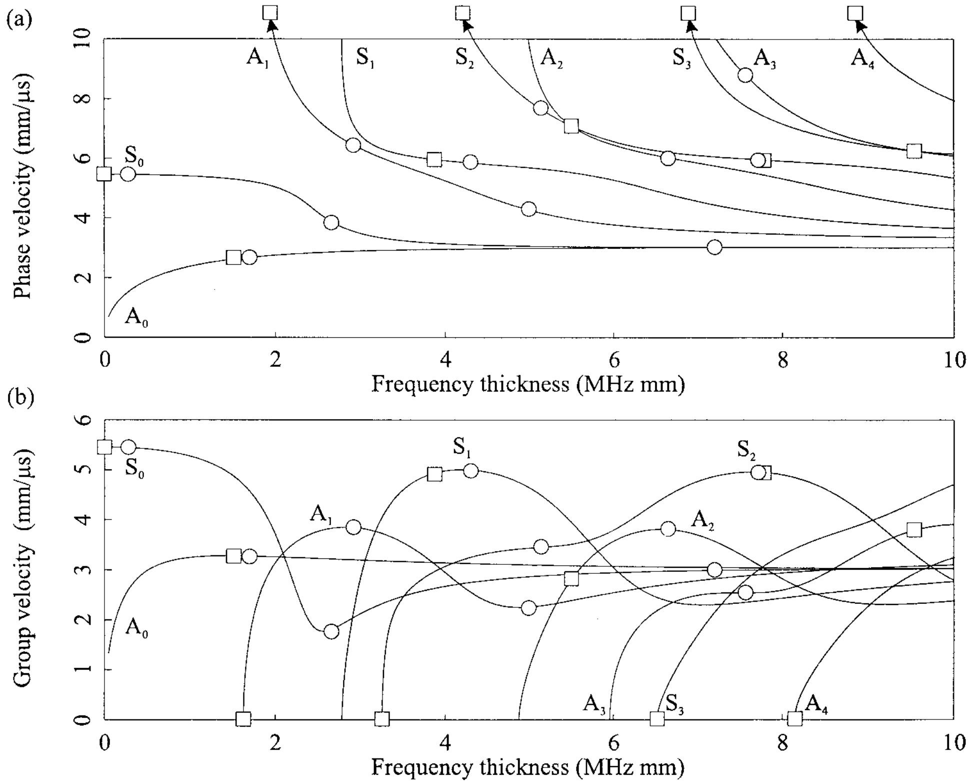

Figure 1. *(a) Phase and (b) group velocity dispersion curves for a steel plate in vacuum. Circles indicate local minima in minimum resolvable distance (MRD) and squares indicate local minima in attenuation for the same plate immersed in water.*

These factors fall into two distinct categories. Factors (i–iii) are properties of the system under inspection, and factors (iv–vi) are determined by the transduction scheme or schemes employed. In the first section of this paper, the properties relating to the system under test are examined, since for a given application these are predetermined, and must therefore dictate the design of the transduction system. The second section of this paper considers the requirements placed on the transduction system, using the angle incidence technique as an example.

SELECTION OF LAMB WAVE MODES

Dispersion

Dispersion is the phenomenon whereby the velocity of a wave mode varies with frequency. It can be seen from Figure 1 that all Lamb wave modes are dispersive. The physical manifestation of dispersion is that when a particular Lamb wave mode is excited by a signal of finite duration (which will be referred to as the input signal), the energy in the Lamb wave spreads out in both space and time as it propagates away from the source. This is illustrated in Figure 2 for the specific example of a five-cycle toneburst with a centre *fd* (frequency thickness product) of 2 MHz mm being input into a steel plate and propagating as the S_0 mode. Figure 2(a) shows the out-of-plane surface displacement of the plate as a greyscale, plotted as a function of distance and time, these two quantities being expressed in terms of the plate thickness, *d*. The input signal is applied at time zero and distance zero. The figure clearly shows the spreading of the propagating signal in distance and time. Experimentally, this will usually be observed as an increase in duration of the received time signal as a receiving transducer is moved away from the source. Three such time-traces are shown in Figure 2 (b–d). It can also be seen from these traces that the increase in duration of the received signal is accompanied by a commensurate reduction in amplitude, which is of course necessary for energy conservation, even in a lossless system.

Both the increase in signal duration and the reduction in amplitude due to dispersion are undesirable in an inspection system. The reduction in amplitude limits the propagation distance that can be achieved before the signal is lost into noise, and the increase in signal duration worsens the resolution that can be obtained. A common practical situation where the latter effect is of great importance occurs when a defect is located in close proximity to another structural feature, such as an edge of a plate or a welded fixture. Even if both the defect and the structural feature cause the incident Lamb wave to be reflected, the defect will only be

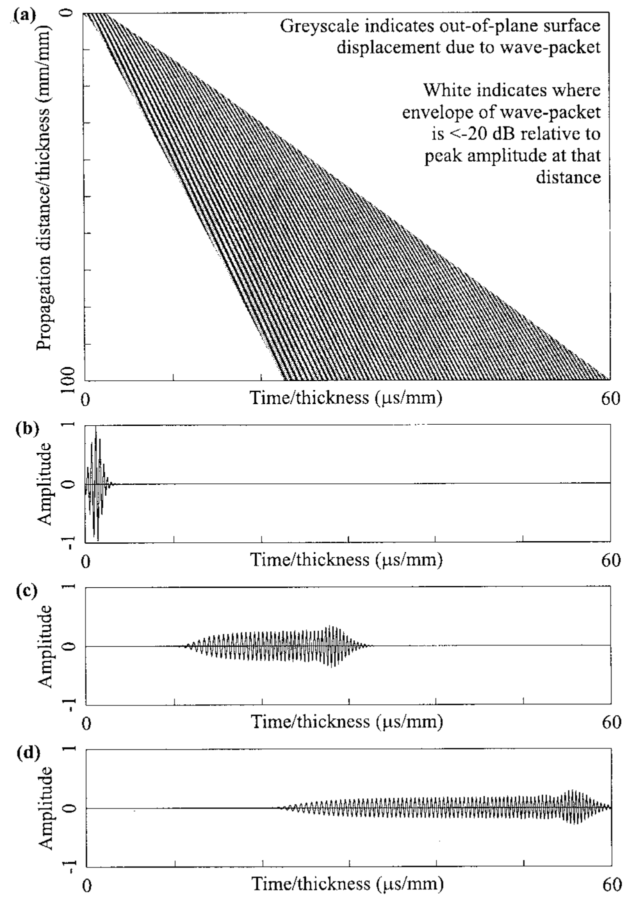

Figure 2. *An example of dispersive wave propagation. (a) Shows a numerical prediction of the spreading of the* S_0 *mode in a steel plate when the input signal is a five-cycle Hanning windowed toneburst with a centre fd at 2.0 MHz mm. On the right hand side are numerical predictions of three time traces that would be received at propagation distance to thickness ratios of (b) 0.1, (c) 50 and (d) 100.*

detected if its reflection can be separately identified. In practice this can only be guaranteed if the two signals are completely separated in time. For this reason, it is desirable to obtain a received signal which is as short as possible. As an example, Figure 3 illustrates the effect of two reflectors generating two signals in a received time-trace. Both reflected signals are assumed to be identical tonebursts with a centre frequency of 1 MHz although they differ in amplitude by a factor of two. In Figure 3(a), the spatial separation of the reflectors is such that the two reflected signals are completely separated in time. As the separation between the reflectors is reduced, the reflected signals begin to overlap, as illustrated in Figure 3(b) and (c) until they become indistinguishable as shown in 3(d). However, if the received signals from the two reflectors were five rather than twenty-cycle tonebursts, then the two signals would be separated in time as shown in 3(e), although the separation of the reflectors is the same as in 3(d).

This simple example shows how the resolution of a long range inspection system is improved if the duration of received signals can be reduced.

Previously, reference has been made (Alleyne and Cawley, 1992a) to certain 'non-dispersive points' existing at the points of stationary group velocity (i.e. $dv_{gr}/df = 0$) on the Lamb wave dispersion curves. This is incorrect, or at least misleading, since even if the group velocity is stationary with respect to frequency at some point, no input signal of finite duration can have an infinitely narrow bandwidth. Hence for testing purposes, all points on the Lamb wave dispersion curves must be considered to be dispersive. The dispersion effect can be minimised by using an input signal with the narrowest bandwidth possible, which is one of the reasons why windowed tonebursts rather than pulses tend to be used as the input signals in Lamb wave applications. For example, an n cycle Hanning windowed toneburst with a centre frequency f_0, has a bandwidth extending from $f_{\min}$ to $f_{\max}$ which is given with reasonable accuracy (the accuracy deteriorates when the number of cycles is low) by:

$$
\begin{aligned}
f_{\min} &= f_0\left(1 - \frac{k}{n}\right) \\
f_{\max} &= f_0\left(1 - \frac{k}{n}\right)
\end{aligned} \tag{1}
$$

where k is a constant, the value of which depends on the definition of bandwidth being used. It can be seen from Equation (1) that increasing the number of cycles in a toneburst reduces its bandwidth. Hence an input signal with a large number of cycles will experience only a small amount of spreading due to dispersion. The duration of a received signal after some particular propagation distance may be regarded as being the sum of two terms, the first, T_{in}, being equal to the duration of the input signal and the second, T_{disp}, being associated with the spreading due to dispersion. Thus while increasing the number of cycles in the input signal (i.e. increasing its duration) will reduce T_{disp}, it will increase T_{in}. Hence in order to minimise the duration of the received signal, a compromise must be made between the bandwidth of the input signal and its duration. In another publication (Wilcox et al., 1999), a practical method for determining the optimum input signal for any point on the Lamb wave dispersion curves and for a given propagation distance is presented. This leads to the result that for every point on the dispersion curves there is a minimum resolvable distance (MRD) or 'best resolution'. The MRD is defined as:

$$
\text{MRD} = \frac{v_0}{d}(T_{\text{in}} + T_{\text{disp}})\Big|_{\min} \tag{2}
$$

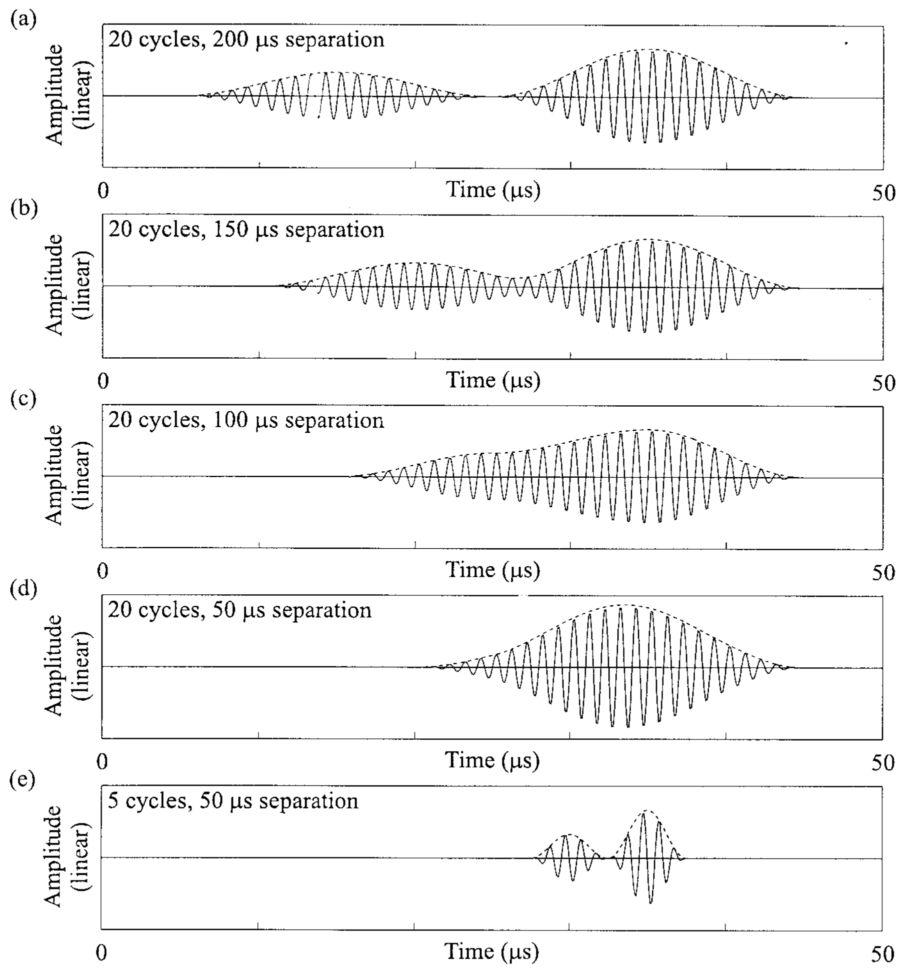

Figure 3. *Predicted time-traces showing (a) two twenty-cycle wave-packets with centre frequencies of 1 MHz completely separated in time. The effect of progressively reducing the separation between centres of the wave-packets is illustrated in time-traces (b), (c) and (d). Time-trace (e) shows the effect of reducing the number of cycles in the wave-packet to five while maintaining the same separation between the centres of the wave-packets as in time-trace (d).*

where v_0 is the group velocity at the centre frequency of the input signal and d is again the plate thickness. The MRD is a measure of the best spatial resolution (as a multiple of the plate thickness) that can be achieved at a point on the dispersion curves if the input signal is optimised. It should be noted that both the optimum number of cycles in an input signal and the MRD are monotonically increasing functions of the propagation distance.

For a particular propagation distance, the MRD may be plotted as a function of frequency for each mode in much the same way as the phase and group velocity dispersion curves are. This is illustrated in Figure 4 for a steel plate and a propagation distance to plate thickness ratio of 800.

For each mode, the MRD is initially high (i.e. the resolution is poor) at low frequencies. This is followed by several maxima and minima before the curves finally monotonically decrease at high frequencies. The latter region corresponds to the velocities of the modes tending towards constant values (equal to the Rayleigh wave velocity for the fundamental modes and the bulk shear wave velocity for all other modes). In terms of long range Lamb wave testing, it is the earlier minima which are of interest, as these correspond to local points of good resolution for each mode. The MRD values at these minima and the corresponding number of cycles in the optimum input signal are tabulated in Table 1. These points are also marked in Figure 1 by circles, and can be seen to be very close to the points of stationary group velocity as expected.

The first thing to observe from Table 1 is that the number of cycles required in the input signal in order to achieve the MRD at the various points ranges from 1 (S_0 at 0.19 MHz mm) to 73 (S_0 at 2.57 MHz mm). It is interesting to observe that the MRD does not improve for higher order modes at higher frequencies. In fact, the lowest MRD (i.e. the best resolution) on any mode is at 1.61 MHz mm on the A_0 mode using a seven-cycle toneburst. This is perhaps surprising since one normally associates higher frequencies with shorter wavelengths and correspondingly better resolution. Although the

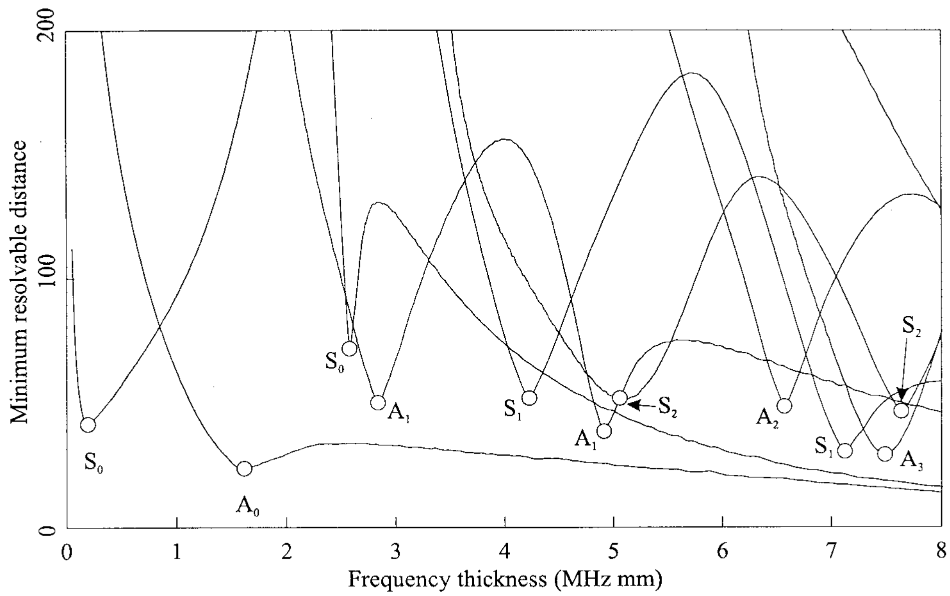

Figure 4. MRD curves for a steel plate and a propagation distance to plate thickness ratio of 800. The circles indicate local minima in the MRD for each mode.

Table 1. Local minima in MRD on the first six Lamb wave modes in a steel plate for a propagation distance of 800d.

Mode	Frequency thickness (MHz mm)	Optimum cycles	MRD
S_0	0.19	1	41
A_0	1.61	7	24
S_0	2.57	73	72
A_1	2.83	25	50
S_1	4.22	31	52
A_1	4.91	56	39
S_2	5.06	40	51
A_2	6.57	59	49
S_1	7.12	19	31
A_3	7.49	67	29
S_2	7.64	51	48

wavelength of Lamb waves does tend to decrease as the frequency is increased, there is no improvement in the resolution which may be obtained, since longer tone-bursts need to be used at high frequencies in order to compensate for generally increasing dispersion effects. A further argument against using higher order modes is that the minima in the MRD curves become increasingly sharp as the order of the mode increases. This means that the centre frequency of operation is very tightly constrained at these points, which is usually not ideal from the point of view of transducer design and operation. This should be compared with the large frequency range over which the A_0 mode has good resolution. Even the minima in the S_0 mode MRD, although it appears sharp in Figure 4, is actually fairly wide compared to the centre frequency. A more general advantage of operating on either of the fundamental modes at frequencies below the cut-off frequency of the A_1 mode (at 1.63 MHz mm) is that only two modes can exist. Hence the problem of preferentially selecting one mode and rejecting others is greatly simplified. This will be addressed in more detail in the second part of the paper.

The preceding discussion has shown that it is desirable to choose an operating point as close as possible to one of the minima in the MRD curves. In order to achieve the MRD, it is necessary to use the correct input signal. Purely from the point of view of achieving the best resolution, there is no advantage in using any operating point other than A_0 at 1.61 MHz mm or S_0 at 0.19 MHz mm.

Attenuation

In the context of Lamb wave testing, attenuation may be defined as the reduction in signal amplitude with propagation distance. In most long range testing applications, the amount of a structure which can be inspected from a single location will be determined by the degree of attenuation of the chosen mode. Hence there is considerable importance in choosing a mode with the lowest attenuation possible.

Lamb wave attenuation can occur by several mechanisms:

(i) signal spreading due to dispersion,
(ii) signal spreading due to beam divergence,
(iii) material damping,
(iv) scattering and
(v) leakage into surrounding media.

The signal spreading due to dispersion causes a reduction in signal amplitude. In practice, since long range testing applications must operate at or close to points of low dispersion, the associated attenuation will be small. Attenuation due to dispersion may be regarded as a spreading of signal energy in the direction of propagation, as opposed to the second mechanism which is signal spreading at right angles to the direction of propagation or beam spreading. In the far field, all Lamb wave transducers have divergent beams, and by energy conservation, it is not to difficult to see that the amplitude of the displacements and stresses in the Lamb waves must decrease in inverse proportion to the square root of the distance from the transducer. The third attenuation mechanism is by energy conversion into heat within the material of the plate due to material damping. For metallic plates, this effect is minimal, but for composite plates, it may be the dominant mechanism for certain modes. The fourth mechanism, scattering, does not in itself generate heat, but causes some of the energy propagating in the mode and direction of interest to be scattered into other directions and possibly other modes. Attenuation due to scattering is complex and may be caused by many factors, such as rough plate surfaces (Chimenti and Lobkis, 1997), inhomegeneities in the plate material, and fixtures attached to the plate (Al-Nasser et al., 1991). Very importantly, scattering is also the mechanism which enables Lamb waves to detect defects (Alleyne and Cawley, 1992b) and in some cases analysis of the scattered signals from defects may enable them to be characterised and distinguished from other features (Cho et al., 1997).

In this paper, attention will be focused on the fifth attenuation mechanism which is by the radiation of acoustic energy into the media surrounding the plate. The degree to which this occurs depends on the degree of coupling between the Lamb wave mode in the plate and the bulk wave modes which can exist in the surrounding media. For a metallic plate in air, this attenuation mechanism is small, due to the large mismatch in acoustic impedance between air and metals. On the other hand, for a plate immersed in liquid, this mechanism can be massive for certain modes, and may well be the deciding factor on the choice of mode.

As an example, the case of a steel plate immersed in water will be considered. The phase and group velocity dispersion curves for this case are almost identical to those shown in Figure 1 for the same plate in vacuum. The attenuation curves for a steel plate in water are plotted in Figure 5. In order for the graph to be independent of the plate thickness, the attenuation has been plotted as attenuation thickness, with units of dB/mm mm. For example, in order to calculate the attenuation per metre in a 2 mm thick plate, the attenuation in dB/mm mm should be multiplied by $1000/2 = 500$ to convert to dB/m.

It can be seen that there are several points where the attenuation on a mode is either a minimum or zero, as indicated by the squares in this figure and in Figure 1. These points are tabulated in Table 2.

From the point of view of long range inspection, several of these points can be discounted immediately, since they occur at or very close to the cut-off frequency of the mode, where the mode is essentially non-propagating. The remaining points of zero attenuation occur when the out-of-plane component of the particle

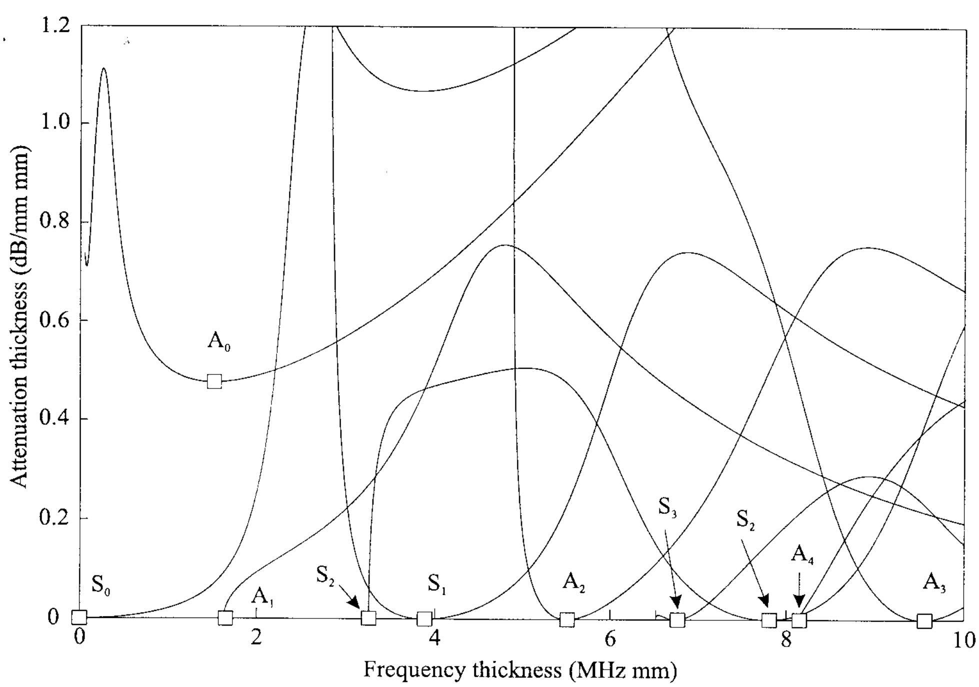

Figure 5. *Attenuation curves for a steel plate in water. The squares indicate local minima in the attenuation for each mode.*

displacement at the surface of the plate is zero. This is because the water is modelled as an ideal inviscid liquid, and can thus support longitudinal but not shear bulk waves. Hence, only the out-of-plane component of the motion of the plate surface couples to the water, and at points on modes where this is zero, the attenuation will also be zero and waves in the plate may propagate without loss.

It should be noted that only a simple leaky case has been discussed here. In cases where a plate is loaded with a viscous fluid, covered in attenuative coatings such as bitumen or even embedded in a solid, the attenuation will generally be much higher (Pavlakovic et al., 1998).

Conclusions on Mode Suitability

This completes the analysis of system properties and it should now be possible to identify one or more possible operating points that satisfy the criteria of attenuation and MRD. For the purposes of the example study presented here, six points have been extracted from Tables 1 and 2, each of which possesses one or both of the attributes of either low attenuation or low MRD. These are listed in Table 3.

A glance at Table 3 indicates at first that the S_0 mode at 0.19 MHz mm is ideal from both the point of view of both having low attenuation and having a low MRD. This is correct, but there is a practical difficulty with using this mode, which is its long wavelength to plate thickness ratio of 26.75. Although it is beyond the scope of the two-dimensional transducer model that will be discussed shortly, there is an important issue concerning the directionality of transducers. In order to generate a well collimated beam of Lamb waves, the width of a transducer (of any type) needs to be five or ten times the wavelength of the Lamb waves (Wilcox et al., 1998). Thus, even on a thin plate, a transducer would need to be very large in order to generate any sort of controlled beam pattern with the S_0 mode at 0.19 MHz mm.

A factor that will not be considered here, but which is of major importance is the sensitivity of the different modes to the defects that it is desired to detect. This is a complex problem due to the number of variables involved and the fact that mode conversion generally occurs. For a given defect geometry and a given incident wave mode, the reflection coefficient can be calculated using finite element techniques (Alleyne and Cawley, 1992b) or other numerical techniques (Pelts et al., 1997). An approximate but not rigorous way of investigating the reflection of guided waves from defects is to look at the distribution of strain energy density (SED) of a mode through the thickness of the plate. In general, modes tend to be most sensitive to defects such as cracks or corrosion which are located at points where the SED is high. On higher order modes, the SED can vary dramatically through the thickness of the plate and must therefore be considered carefully. For the S_0 mode, the SED is approximately uniform through the thickness of the plate at low frequencies, so this mode should exhibit reasonable sensitivity to defects (but incidentally not to delaminations (Guo and Cawley, 1993)) anywhere in the thickness. The A_0 mode has an SED which is higher at the plate surface, so this mode should be sensitive to common defects such as surface breaking cracks or corrosion. This is another argument in favour of using one of the fundamental modes.

Table 2. Local minima in attenuation on the first six Lamb wave modes in a steel plate in water.

Mode	Frequency thickness (MHz mm)	Attenuation thickness (dB/mm mm)	Comments
S_0	$\to 0$	$\to 0$	
A_0	1.52	0.48	The lowest attenuation possible on A_0 mode
A_1	1.63	$\to 0$	Zero attenuation occurs as mode approaches cut off frequency
S_2	3.26	$\to 0$	Zero attenuation occurs as mode approaches cut off frequency
S_1	3.89	0	
A_2	5.51	0	
S_3	6.75	0	Zero attenuation occurs very close to cut off frequency
S_2	7.78	0	
A_3	9.54	0	

Table 3. Some possible operating points on the first six Lamb wave modes.

Mode	Frequency thickness (MHz mm)	Optimum cycles	MRD	Attenuation thickness (dB/mm mm)	Wavelength/ thickness (mm/mm)	Comments
S_0	0.19	1	41	0.0006	26.75	Low MRD and low attenuation
A_0	1.61	7	24	0.48	1.65	Lowest MRD, but high attenuation
S_1	3.89	42	94	0	1.53	No attenuation, but high MRD
A_1	2.83	25	50	0.196	2.13	Moderate MRD, moderate attenuation
S_2	7.78	54	56	0	0.77	No attenuation, moderate MRD
A_2	5.51	202	202	0	1.28	No attenuation, but high MRD

TRANSDUCER DESIGN

Having assessed which Lamb wave mode or modes are suitable for a particular inspection task, it is next necessary to consider the transduction requirements. A good Lamb wave transduction scheme must be modally selective as well as simply being able to transmit and receive Lamb waves. The first way in which Lamb wave transduction schemes achieve modal selectivity is by only operating over a limited frequency bandwidth, as discussed in the first part of this paper. However, reference to the dispersion curves shown in Figure 1, shows that frequency selectivity alone is not enough to select a single mode, hence the need for some further means of selectivity. In most practical transduction schemes the second degree of modal selectivity is achieved by the transmitting transducer applying a spatially periodic distribution of surface tractions on the surface of the plate.

Many different types of Lamb wave transduction schemes exist such as angle-incidence methods (Viktorov, 1967 and Jia, 1996), interdigital transducers (IDTs) (Monkhouse et al., 1997 and Demol et al., 1996), Hertzian contact transducers (Degerketin and Khuri-Yakub, 1996), electro-magnetic acoustic transducers (EMATs) (Alers and Burns, 1987) etc.

Here, the angle incidence technique will be used as an example of the procedure by which transducer performance can be assessed, although the possibility of extending the procedure to other transduction schemes should be clear. In an angle incidence configuration, a conventional bulk wave transducer is positioned at an angle above the surface of the plate under inspection as shown in Figure 6(a). The space between the transducer and the plate is filled with a coupling medium, such as water or in some cases, perspex. When used to transmit, the transducer excites plane bulk compressional waves in the coupling medium that impinge on the surface of the plate at an angle. This causes a spatially and temporally periodic distribution of out-of-plane surface stresses or tractions to be set up on the surface of the plate. It will be shown in the following analysis that the spatial periodicity of the surface tractions causes the angle incidence technique to preferentially excite Lamb waves with a certain phase velocity. When used as a receiver, the process is reversed: when a Lamb wave in the plate passes under the coupling medium it radiates energy into it in the form of bulk compressional waves. The angle at which the bulk waves propagate at is determined by the phase velocity of the Lamb wave and the bulk compressional wave velocity in the coupling medium. The receiving bulk wave transducer will be most sensitive to bulk waves that are propagating at the same angle to that at which it is inclined.

Excitability and Detectability of Lamb Waves

In the angle incidence configuration, the transmitter is assumed to apply only out-of-plane surface tractions

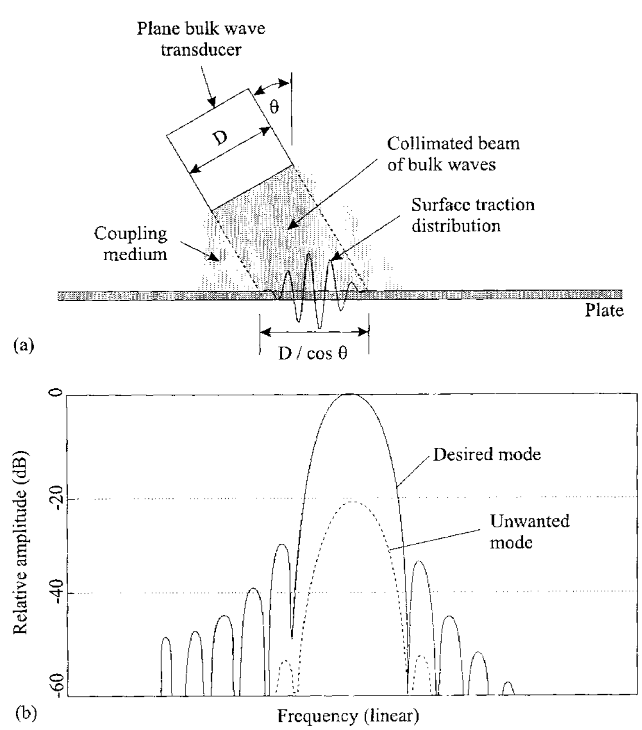

Figure 6. (a) Schematic diagram showing an angle incidence transduction configuration when used as a transmitter an (b) a typical modal selectivity plot.

to the surface of the plate and the receiver is assumed to be sensitive only to out-of-plane displacements on the plate surface. The first step in the transducer model used here is therefore to compute the amplitude of out-of-plane surface displacement at a point on a plate which is caused by the application of an out-of-plane force at another point on the plate. Once this is known, the principle of superposition can be used to calculate the performance of transmitting and receiving transducers of finite areas. Throughout this analysis, the Lamb wave system is assumed to be two-dimensional and in a plain-strain state. This is consistent with the boundary conditions that are applied in order to compute the familiar Lamb wave dispersion curves, such as those shown in Figure 1. The direction x will be taken as the direction of propagation of the Lamb waves.

Consider an out-of-plane time harmonic force, $Fe^{-i\omega t}$, being applied at a single a point on the surface of the plate in the plane-strain two-dimensional model, where ω is the angular frequency and t is time. This force will excite, to varying degrees, all of the Lamb wave modes which can exist at the excitation frequency. Each of these modes will be associated with a temporally and spatially harmonic out-of-plane surface displacement on the surface of the plate. The ratio between the amplitude of the out-of-plane surface displacement of the nth mode to the amplitude of the out-of-plane excitation force, F, may be defined as the out-of-plane excitability, $E_n(\omega)$, of the mode, so that:

$$u(x,t) = FE_n(\omega)e^{i(k_n\omega)x-\omega t} \tag{3}$$

where $k_n(\omega)$ is the angular wavenumber of that mode and x is the distance from the excitation point. The excitability can be computed, using the method of Viktorov (1967), and plotted as a function of frequency to yield the out-of-plane excitability curves for the plate. These are plotted for a steel plate in Figure 7.

The similarities between the excitability curves shown in Figure 7 and and the leakage attenuation curves for a plate in water shown in Figure 5 are obvious, and there is a simple explanation for this. Leakage attenuation is caused by the coupling between out-of-plane surface displacement of Lamb waves in the plate and bulk waves in the surrounding fluid. Similarly, excitability is effectively a measure of the coupling between an external out-of-plane force and the out-of-plane surface displacement of Lamb waves in the plate. Hence modes with a small amount of out-of-plane surface displacement have low leakage attenuation and are also hard to excite with an out-of-plane surface force. For this reason alone it would be expected that the angle incidence technique is not a good transduction method to use with low attenuation Lamb wave modes, and this will shortly be demonstrated quantitatively.

Transducer Model

When a transmitting transducer is excited by a continuous sinusoidal voltage, $V_{\text{in}}(\omega)e^{-i\omega t}$, a spatially and temporally harmonic distribution of out-of-plane surface tractions $\sigma(x,t)$ is set up over a finite region of length c on the surface of the plate. It is assumed that this region is centred about a point at position x_1 and

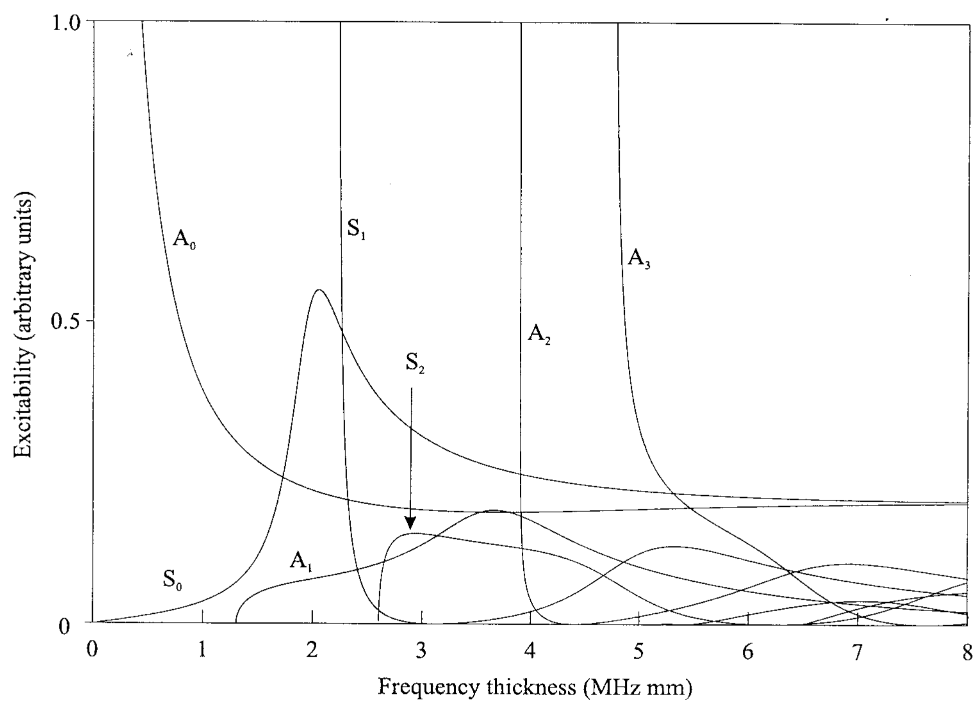

Figure 7. *Out-of-plane excitability curves for a steel plate.*

therefore extends from $x_1 - c/2$ to $x_1 + c/2$. The amplitude of the surface tractions within this region is determined by an aperture function, $w(x - x_1)$, which for the purposes of this study will be assumed to have a Hanning window shape. Hence the aperture function, w, is defined as:

$$w(x) = \begin{cases} \frac{1}{2}\left(\cos\left(\frac{2\pi x}{c}\right) + 1\right) & |x| \leq c/2 \\ 0 & |x| > c/2 \end{cases} \qquad (4)$$

Thus the surface traction distribution on the surface of the plate is given by:

$$\sigma(x, t) = aV_{\text{in}}(\omega)w(x - x_1)e^{ik_0(\omega)(x-x_1)} \qquad (5)$$

where a is a constant of proportionality that represents the conversion of input voltage to surface traction by the transducer and $k_0(\omega)$ is the spatial period of the surface tractions. Using the definition of the excitability function given in Equation (3), an expression can now be written for the out-of-plane surface displacement at any position on the plate:

$$\begin{aligned} u(x,t) &= \int_{x_1-c/2}^{x_1+c/2} \sigma(\chi, t)E_n(\omega)e^{i(k_n(\omega)(x-\chi)-\omega t)}d\chi \\ &= E_n(\omega)aV_{\text{in}}(\omega)e^{i(k_n x-\omega t)}e^{-ik_n x_1} \\ &\quad \int_{x_1-c/2}^{x_1+c/2} w(\chi - x_1)e^{i(k_0(\omega)-k_n(\omega))(\chi-x_1)}d\chi \end{aligned} \qquad (6)$$

where χ is a temporary variable that is used to integrate the contributions from the surface tractions over the area of the transducer. By making the substitution $\phi = \chi - x_1$, the integral in Equation (6) can be written as a function of the wavenumber of the propagating mode, $k_n(\omega)$, the wavenumber associated with the transducer, $k_0(\omega)$, and the aperture function $w(\phi)$:

$$T(k_n(\omega), k_0(\omega), w) = \int_{-c/2}^{c/2} w(\phi)e^{i(k_0(\omega)-k_n(\omega))\phi}d\phi \qquad (7)$$

$T(k_n(\omega), k_0(\omega), w)$ can be regarded as a function of the sensitivity of the transducer to the wavenumber of Lamb waves in the plate.

Equation (6) can now be written as:

$$\begin{aligned} u(x,t) &= aE_n(\omega)V_{\text{in}}(\omega)e^{i(k_n(\omega)x-\omega t)}e^{-ik_n(\omega)x_1} \\ &\quad \times T(k_n(\omega), k_0(\omega), w) \end{aligned} \qquad (8)$$

The receiving transducer is usually made to be sensitive to a similar spatial distribution of surface displacements in the plate in order to obtain preferential reception of the same mode. Consider a receiving transducer, identical to the transmitting transducer, which is sensitive to surface displacements centred at a location x_2. The electrical output, $V_{\text{out}}(\omega)$, from the receiving transducer may be expressed as the integral of the out-of-plane surface displacement of the plate multiplied by its aperture function, $w(x - x_2)$, and a spatially harmonic function with period $k_0(\omega)$ to give:

$$V_{\text{out}}(\omega) = b\int_{x_2-c/2}^{x_2+c/2} u(x,t)e^{ik_0(\omega)(x-x_2)}w(x - x_2)\,dx \qquad (9)$$

where b is again a constant of proportionality for the transducer that this time represents the conversion of surface displacement to output voltage. Substitution of the expression given in Equation (8) for $u(x, t)$ gives:

$$\begin{aligned} V_{\text{out}}(\omega) &= abT(k_n, k_0, w)V_{\text{in}}(\omega)E_n(\omega)e^{i(k_n(\omega)(x_2-x_1)-\omega t)} \\ &\quad \int_{x_2-c/2}^{x_2+c/2} w(x - x_2)e^{i(k_n(\omega)+k_0(\omega))(x-x_2)}dx \end{aligned} \qquad (10)$$

By making the substitution $\phi = x - x_1$, the integral can be made equal to that given in Equation (7) so that finally:

$$\begin{aligned} V_{\text{out}}(\omega) &= abV_{\text{in}}(\omega)e^{i(k_n(\omega)(x_2-x_1)-\omega t)}E_n(\omega) \\ &\quad \times (T(k_n, k_0, w))^2 \end{aligned} \qquad (11)$$

The exponential term in this equation is the propagation term, and gives the phase shift of the Lamb waves due to propagation between x_1 and x_2. However, this does not affect the amplitude of V_{out}. The next term is the frequency dependent excitability of the Lamb wave mode, which is a property of the plate itself and independent of the transduction system. The transducer selectivity appears only in the final term, $T(k_n(\omega), k_0(\omega), w)$. This term is squared because the transmitting and receiving transducers both have the same sensitivity function.

For inter-digital and comb type transducers, the spatial periodicity, k_0, is fixed for all frequencies. Since the length of such a device and therefore the length, c, of the aperture function, w, are also constant, the selectivity function of such a device will be a constant function of wavenumber for all frequencies. If the peak of this function for a particular transducer is plotted on the phase velocity dispersion curves for a plate, it will appear as a diagonal line, referred to as the operating line, through the origin. The gradient of this line is equal to $2\pi/k_0$ (i.e. a line of constant wavelength as noted by Monkhouse et al., 1997).

In the angle incidence configuration, it can be shown (see for example Ditri and Rose, 1994) that:

$$k_0 = \frac{\omega}{v_w}\sin\theta \qquad (12)$$

where v_w is the velocity of sound in the coupling medium and θ is the transducer inclination angle. Assuming that the spatial length, c, of the aperture function, w, is again independent of frequency, the selectivity function T becomes a function of phase velocity, which is constant for all frequencies. The peak sensitivity occurs when at a phase velocity v_0, given by:

$$v_0 = \frac{v_w}{\sin\theta} \tag{13}$$

Hence, an angle incidence configuration of transducers can be regarded as having a horizontal operating line on the phase velocity dispersion curves.

Selectivity of Transducers

The model described in the previous subsection was used to investigate the performance of the angle incidence technique for exciting and receiving Lamb waves at each of the candidate operating points listed in Table 3. The procedure to obtain results from the model for a particular operating point is as follows. First, the centre frequency and number of cycles in the input signal is specified according to the values given in Table 3. From this $V_{\mathrm{in}}(\omega)$ is obtained by Fourier transform. Next the required parameters for the transmitting and receiving transducers (assumed identical) are specified. As shown in Figure 6(a), the parameters for the angle incidence technique are the diameter of the bulk wave transducer, D, its incident angle, θ, and the bulk wave velocity in the coupling medium. The incident angle, θ, is calculated from Equation (13), so that the peak sensitivity occurs at the phase velocity of the mode of interest at the centre frequency of the input signal. The spatial region of surface traction on the surface of a plate that is associated with an angle incidence configuration is assumed to extend over a distance $D/\cos\theta$, as indicated in Figure 6(a). This corresponds to the projection of the transducer diameter onto the surface of the plate. It is over this region that a Hanning window aperture function is applied, hence $c = D/\cos\theta$ in Equation (4). The amplitude of $V_{\mathrm{out}}(\omega)$ for each mode is then calculated separately as a function of frequency using Equation (11). From this a modal selectivity plot similar to that shown in Figure 6(b) can be produced. The modal selectivity can be defined as the ratio between the amplitude of the desired mode at the centre frequency of the input signal to the peak amplitude of the next most strongly excited mode over the bandwidth of the input signal. Hence, in the example shown in Figure 6(b), a modal selectivity of 20 dB has been acheived.

For each of the points listed in Table 3, the model was run with increasing diameters of transducer until a modal selectivity of 20 dB was reached. The transducer sizes thus calculated are tabulated in Table 4. In some cases, it is not possible to achieve the desired level of modal selectivity at all. This can occur if the phase velocity of the desired mode at the centre frequency of the input signal is equal to that of an undesired mode at any point within the bandwidth of the input signal. In this situation, the angle incidence technique will be equally sensitive to both modes. Practically, the only way to suppress the unwanted modes at these points is to reduce the bandwidth of the input signal.

Table 4. Angle incidence transducer specifications required to achieve 20 dB signal to coherent noise ratio at the operating points listed in Table 3.

Mode	Frequency thickness (MHz mm)	Transducer angle θ (°)	Transducer diameter D/thickness (mm mm)
S_0	0.19	16	55
A_0	1.61	34.5	2.5
S_1	3.89	14.6	–
A_1	2.83	13.1	65
S_2	7.78	14.6	–
A_2	5.51	12.2	–

Dashes indicate that the mode could not be excited or detected at the required frequency using the angle incidence method.

It can be seen from Tables 3 and 4 that at the points where the angle incidence technique cannot be used, the attenuation is zero. This is to be expected, since the angle incidence technique relies on coupling between bulk waves in the coupling medium and Lamb waves in the plate, which is also what defines the attenuation. Even at the 'compromise' points on S_0 at 0.19 MHz mm and A_1 at 2.83 MHz mm where the attenuation is low but not zero, the diameters of transducers required are very large in order to achieve the required modal selectivity. Although it may be possible to construct and use transducers with diameters in excess of 50 times the plate thickness on 1 mm thick plates, they are clearly not a feasible option on, say, 10 mm thick plates. In fact the only practical application of the angle incidence technique for long range testing is for modes which have high attenuation in liquid loaded plates, such as the A_0 mode. This is not to say that such modes have no use; in a gas filled pressure vessel for example where the attenuation due to leakage will be low, using the A_0 mode is by far the simplest option.

In order to use a low attenuation mode, alternative transduction schemes must be investigated. One possibility is to use EMATs (Alers and Burns, 1987), which can potentially couple to in-plane motion at the surface of a plate. Another alternative is to use shearing (rather than expanding) piezoelectric devices (Alleyne and Cawley, 1996) in direct contact with the surface of the plate. Clearly, the transducer model described here could be expanded to model these types of devices if a complementary in-plane excitability function was derived.

CONCLUSIONS

This paper has presented a systematic procedure for mode selection and transducer design in long range guided wave inspection systems. The most important aspect of the procedure is the separation of the properties of the structure under inspection from the properties of the transduction system, since the former always dictates the latter. Hence the first step when designing a guided wave inspection system for a particular task is to analyse the structure itself in order to identify one or more candidate operating points on the guided wave modes that may propagate in the structure.

The number of criteria by which candidate operating points are selected depends on the nature of the inspection task. Two fundamental criteria that always need to be considered are resolution and sensitivity to the defects to be detected. A third criterion that must be considered in the majority of practical situations is the attenuation of the guided waves since this will be a significant factor in determining the amount of a structure that can be inspected from a single location. It should be noted that the resolution and attenuation for any operating point can be obtained from the dispersion curves for a structure. On the other hand, the sensitivity to defects must be calculated using more time-consuming techniques such as finite element analysis or analytic methods. For this reason, it makes sense to examine resolution and attenuation first in order to obtain a provisional list of operating points at which sensitivity studies can be performed.

In the simple case of an isotropic plate that has been considered here, the points of highest resolution were identified first. This yields two candidate operating points, one on A_0 at its point of minimum dispersion and one on S_0 at low frequency. Interestingly, from the point of view of resolution, there is no advantage in considering operating points at higher frequencies on higher order modes. If the plate is liquid loaded (as, for example, in the case of the wall of a storage tank) the attenuation due to leakage must be considered. If this is the case then the A_0 operating point becomes unsuitable as the attenuation is very high, but the attenuation is low at the S_0 operating point. The sensitivity to defects at either of these operating points has not been considered here, but in this case, the information is available in the literature (Lowe, 1998; Lowe et al., 2000). In practice, the sensitivity at both points to most simple defects, such as cracks and notches, will be adequate and of similar magnitude due to the fairly uniform distribution of energy through the thickness of the plate. Where the sensitivity considerations become much more important is in anisotropic materials. One example of this is delamination detection in composites where certain modes have been found to be completely blind to delaminations at particular depths. It should be noted that composites provide several other complications that need to be considered when selecting candidate operating points. For instance, the guided wave propagation characteristics will be different in different directions (Simon et al., 1997). Also, material damping is significant in composites and this will cause varying attenuation of the guided wave modes even in a free plate. This has been shown by Chan and Cawley (1998) for the more simple case of an isotropic free plate with viscoelastic damping.

Once a number of candidate operating points different transduction schemes can be examined. Although the assessment criteria for transduction schemes are fairly straightforward to define, the number of possible transduction schemes is continuously increasing. At some operating points, there may be a number of suitable transduction methods while at others there may be none that yet exist. For these reasons the selection of the actual transduction method is open-ended. However, given a particular transduction method and a reasonable model of how it operates, it is possible to then determine whether or not it is suitable for use at a given operating point.

In the example used in this paper, the feasibility of using the angle incidence technique at the various candidate operating points has been investigated. This has been used to demonstrate that this transduction technique is never suitable for use at any of the operating points where the attenuation is low.

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